

## Bilateral Mass Comparison between INMS/NRC (Canada) and LACOMET (Costa Rica), SIM.7.36

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### ABSTRACT

A bilateral comparison was organized in the context of a CIDA (Canadian International Development Agency) project between INMS/NRC (Canada) and LACOMET, the national metrology laboratory of Costa Rica. This comparison will provide to LACOMET part of the technical support for their accreditation scopes under ISO 17025 and the submission of Mass CMCs in the context of the CIPM - MRA. The pilot laboratory is NRC. The nominal values of the mass standards used for the comparison were 1 kg, 200 g, 20 g, 2 g and 200 mg. The measurements were done between fall 2005 and spring 2006. The degrees of equivalence resulting from this comparison are within their respective standard uncertainties, showing that the comparison is a success for each weight and each laboratory.

### 1. INTRODUCTION

A bilateral comparison was organized in the context of a CIDA (Canadian International Development Agency) project between NRC, Canada and LACOMET, Costa Rica. This comparison will provide to LACOMET part of the technical support for their accreditation scopes under ISO 17025 and the submission of Mass CMCs in the context of the CIPM - MRA. The pilot laboratory is NRC. The nominal values of the mass standards used for the comparison were 1 kg, 200 g, 20 g, 2 g and 200 mg. The measurements were done between fall 2005 and spring 2006.

### 2. CONTEXT OF THE PROJECT

A proper national measurement system is vital in order for a country to meet the challenges and requirements of local and foreign markets. The concept of measurement traceability is a key one for understanding how a properly organized national measurement system provides greater assurance of accurate measurements. At one end of the traceability chain are the routine measurements related to manufacturing, commerce and environmental protection. At the other end are the national metrology institutes (NMIs), like LACOMET and NRC, responsible for the most accurate possible realization of the SI units of measurement (the kilogram, the meter, the second, the volt, etc.) and their dissemination.

In order to facilitate acceptance of results from a measurement laboratory in one economy by an end user in another economy, the NMIs of all developed economies devote a substantial fraction of their resources to participation in high-level measurement comparisons with other NMIs. These comparisons are intended to demonstrate a high degree of equivalence of national measurement standards. Once this demonstration of equivalence is accomplished, the traceability chains established independently by each NMI provide a basis for acceptance of routine measurement results made in one country by an end user in another country.

A project supported by the Canadian International Development Agency (CIDA) has been proposed and executed in order to bring LACOMET to the level of this bilateral comparison with NRC by training its personnel in Canada and make sure the equipment of LACOMET is at the proper level of quality for the requirements of Costa Rica.

### 3. WEIGHTS

The weights of the comparison belong to NRC. Their nominal value, identification, and density are shown in Table 1.

The magnetic properties of the 1 000 g and the 200 g weights were measured and found to conform to the requirements of OIML R111 E1 weights. The other weights are too small to have their magnetic properties measured.

Table 1. Weights used for the comparison.

Nominal Value / g	ID	Density / kg·m <sup>-3</sup>	u (Density) / kg·m <sup>-3</sup>
1000	CLASkg	7 902	75
200	200CLAS	7 678	138
20	20D4	7 850	20
2	2D4	7 850	20
0.2	0.2D4	7 850	20

**3.1. Dates of Calibration**

The dates the calibrations were performed at each laboratory are shown in Table 2.

Nominal Value / g	ID	NRC 2005-11	LACOMET 2006-03	NRC 2006-05
1000	CLASkg	2005-11-03	2006-03-14	2006-05-23
200	200CLAS	2005-11-11	2006-03-14	2006-05-24
20	20D4	2005-11-07	2006-03-14	2006-06-02
2	2D4	2005-11-07	2006-03-14	2006-06-02
0.2	0.2D4	2005-11-04	2006-03-14	2006-06-05

**3.2. Stability of the Weights**

As described below in the section Reference values, the second measurement at NRC, identified as NRC 2006-05, was made to check the stability of the weights. The results below show clearly that the weights have been stable during the period of the comparison.

**4. METHODOLOGY OF THE MEASUREMENTS**

The measurements of each weight were performed applying orthogonal design calibration procedures in both laboratories [3]. For the kilogram weight, the measurement method applied was direct comparison with double substitution RTTR.

**5. REFERENCE VALUES**

The reference values were calculated as weighted means between NRC and LACOMET. The NRC values were simple means taken from the two sets of measurements, in November 2005 and May 2006, with uncertainties as the means of the respective uncertainties, so as to make the two sets of measurements equivalent to one single set with no preference to one or the other. The purpose of the second measurement in May 2006 is to check the stability of the standards. Instead of arbitrarily choosing one or the other NRC series of measurements, simple means were calculated as:

$$M_{NRC} = \frac{M_{NRC-2005} + M_{NRC-2006}}{2} \tag{1}$$

$$u_{NRC} = \frac{u_{NRC-2005} + u_{NRC-2006}}{2}$$

For the Reference Value RV:

$$RV = \frac{\frac{M_{NRC}}{u_{NRC}^2} + \frac{M_{LACOMET}}{u_{LACOMET}^2}}{\frac{1}{u_{NRC}^2} + \frac{1}{u_{LACOMET}^2}} \tag{2}$$

with an uncertainty given by:

$$u_{RV}^2 = \frac{1}{\frac{1}{u_{NRC}^2} + \frac{1}{u_{LACOMET}^2}} \tag{3}$$

from which we calculate the Expanded Uncertainty  $U_{RV}$  as:

$$U_{RV} = 2u_{RV} \tag{4}$$

**6. RESULTS**

**6.1. Results Reported**

The results reported are shown in Table 3, below.

**6.2. Reference Values**

The reference values are shown in Table 4.

Table 4. Reference Values.

Nominal Value / g	Identification	RV	
		Value / mg	U /mg
1000	CLASkg	2.074	0.055
200	200CLAS	0.107	0.024
20	20D4	0.0027	0.0064
2	2D4	0.0279	0.0048
0.2	0.2D4	-0.0007	0.0011

**7. DEGREES OF EQUIVALENC**

The degrees of equivalence, shown in Table 5, are expressed with two numbers as: 1) the difference between the laboratory values and the reference values and 2) the respective expanded uncertainties of these differences for each participant. Here is how the uncertainties are calculated:

Table 3. Results reported.

Nominal Value / g	Identification	NRC 2005-11		LACOMET 2006-03		NRC 2006-05		NRC (means)	
		Value / mg	u / mg	Value / mg	u / mg	Value / mg	u / mg	Value / mg	u / mg
1000	CLASkg	2.071	0.025	2.20	0.20	2.073	0.031	2.072	0.028
200	200CLAS	0.11	0.01	0.02	0.16	0.102	0.012	0.107	0.012
20	20D4	0.0024	0.0034	-0.001	0.010	0.0039	0.0034	0.0032	0.0034
2	2D4	0.0272	0.0025	0.0303	0.0086	0.0282	0.0025	0.0277	0.0025
0.2	0.2D4	-0.00101	0.00065	0.00042	0.00094	-0.00156	0.00065	-0.00128	0.00065

Table 5. Degrees of equivalence.

Nominal Value / g	Identification	NRC - RV		LACOMET - RV		LACOMET - NRC	
		DoE / mg	U / mg	DoE / mg	U / mg	DoE / mg	U / mg
1000	CLASkg	-0.0025	0.0078	0.13	0.40	0.13	0.40
200	200CLAS	0.0005	0.0018	-0.09	0.31	-0.09	0.31
20	20D4	0.0005	0.0023	-0.004	0.018	-0.004	0.020
2	2D4	-0.0002	0.0014	0.002	0.017	0.003	0.018
0.2	0.2D4	-0.00055	0.00074	0.0012	0.0015	0.0017	0.0023

$$u^2 \{M_{NRC} - RV\} = u_{NRC}^2 + u_{RV}^2 - 2\sigma_{NRC-RV}^2$$

$$u^2 \{M_{LACOMET} - RV\} = u_{LACOMET}^2 + u_{RV}^2 - 2\sigma_{LACOMET-RV}^2$$

(5)

Since the Reference Value *RV* is covariant with  $M_{NRC}$  and  $M_{LACOMET}$ , covariances  $\sigma_{NRC-RV}^2$  and  $\sigma_{LACOMET-RV}^2$  must be included in the calculation of the uncertainties. Eq. (5) can be written as (see Appendix for demonstration) and Cox [1]:

$$u^2 \{M_{NRC} - RV\} = \frac{\frac{u_{NRC}^2}{1} - \frac{u_{LACOMET}^2}{1}}{\frac{1}{u_{NRC}^2} + \frac{1}{u_{LACOMET}^2}} = u_{NRC}^2 - u_{RV}^2$$

$$u^2 \{M_{LACOMET} - RV\} = \frac{\frac{u_{LACOMET}^2}{1} - \frac{u_{NRC}^2}{1}}{\frac{1}{u_{NRC}^2} + \frac{1}{u_{LACOMET}^2}} = u_{LACOMET}^2 - u_{RV}^2$$

(6)

The degrees of equivalence between the two laboratories LACOMET – NRC is simply the difference between the values of the two

laboratories with the expanded square root of the quadratic sum of their uncertainties as the uncertainties of the difference, since the respective uncertainties of the laboratories are not correlated.

### 8. CONCLUSION

All the absolute values of the differences (DoE) are smaller than their respective expanded uncertainties. This indicates that the comparison results were successful for each weight and each laboratory. This means also that LACOMET (Costa Rica) claims for their accreditation scope in mass and corresponding CMCs are supported by the results of this comparison.

### REFERENCES

- [1] Cox M. G., *Metrologia*, 2002, 39, 589-595, *The evaluation of key comparison data*.
- [2] Bevington, P. R., *Data Reduction and Error Analysis for the Physical Sciences*, McGraw-Hill, 1969.
- [3] Chapman, G. D., *Orthogonal Designs for Calibrating Kilogram Submultiples*, NRCC 25819, 2004.

**APPENDIX**

We have to prove, according to Bevington [2] and the GUM, that:

$$\begin{aligned}
 u^2 \{M_{NRC} - RV\} &= u_{NRC}^2 + u_{RV}^2 + 2\sigma_{NRC-RV}^2 \frac{\partial \{M_{NRC} - RV\}}{\partial M_{NRC}} \frac{\partial \{M_{NRC} - RV\}}{\partial RV} \\
 &= \frac{u_{NRC}^2}{\frac{1}{u_{NRC}^2} + \frac{1}{u_{LACOMET}^2}} \\
 &= u_{NRC}^2 - u_{RV}^2
 \end{aligned} \tag{7}$$

The first step is to express the partial derivatives:

$$\begin{aligned}
 \frac{\partial \{M_{NRC} - RV\}}{\partial M_{NRC}} &= 1 \\
 \frac{\partial \{M_{NRC} - RV\}}{\partial RV} &= -1
 \end{aligned} \tag{8}$$

so that

$$u^2 \{M_{NRC} - RV\} = u_{NRC}^2 + u_{RV}^2 - 2\sigma_{NRC-RV}^2 \tag{9}$$

Since  $RV$  depends on  $M_{NRC}$ , we have:

$$\sigma_{NRC-RV}^2 = u_{NRC}^2 \frac{\partial RV}{\partial M_{NRC}}$$

$$\begin{aligned}
 &\frac{\partial \left\{ \frac{M_{NRC} + M_{LACOMET}}{u_{NRC}^2 + u_{LACOMET}^2} \right\}}{\partial M_{NRC}} \\
 &= u_{NRC}^2 \frac{\left\{ \frac{1}{u_{NRC}^2} + \frac{1}{u_{LACOMET}^2} \right\}}{\frac{1}{u_{NRC}^2} + \frac{1}{u_{LACOMET}^2}} \\
 &= \frac{1}{\frac{1}{u_{NRC}^2} + \frac{1}{u_{LACOMET}^2}} \\
 &= u_{RV}^2
 \end{aligned} \tag{10}$$

Combining this last result with Eq. (9), we obtain:

$$\begin{aligned}
 u^2 \{M_{NRC} - RV\} &= u_{NRC}^2 + u_{RV}^2 - 2\sigma_{NRC-RV}^2 \\
 &= u_{NRC}^2 - u_{RV}^2 \\
 &= u_{NRC}^2 - \frac{1}{\frac{1}{u_{NRC}^2} + \frac{1}{u_{LACOMET}^2}} \\
 &= \frac{u_{NRC}^2}{\frac{1}{u_{NRC}^2} + \frac{1}{u_{LACOMET}^2}} \tag{11}
 \end{aligned}$$

This proves the first relation of Eq. (6). The second relation in Eq. (6) can be proved following the same way, by exchanging “NRC” and “LACOMET” in Eqs. (7) to (11).